AVS-48 Picobridge® Project

Five-segment square root approximation

INTRODUCTION

It can be sufficient to approximate an analog square-root function cheaply by a few linear segments in applications, where the function needs not be very accurate. Linearisation of a temperature controller's heating power vs. its PID output [$P_H = R_H * I^2$] is an example. In a closed-loop control system, accuracy is mainly determined by circuits other than the power output stage. Without such a linearisation, the closed-loop gain will change with the sample's heat load [$P_H = R_H * (I + I_{LOAD})^2$]. This can make it more difficult to optimise the PID settings in terms of speed and stability. The following simple circuit approximates the function

(1) $V_{OUT} = 3 * \text{sqrt}(V_{IN}) / \text{sqrt}(3)$

using one quad and one single low-cost operational amplifier plus 12 resistors.

DESIGN GOAL

The operational amplifiers in our AVS-48 will be powered with +/- 5 Volts and therefore all signal voltages must be limited to +/- 3 Volts. The square-root circuit is placed after the PID section, which delivers +3V at maximum, and before the heater power stage, which requires +3V for maximum output current.

Figure 1 shows sqrt(V_{IN}) where 0.. V_{IN}..+3V (curve a). When sqrt() is approximated by linear segments, each segment line has a smaller slope than its predecessor. We need an attenuator, whose attenuation increases stepwise at each corner point for input voltages that rise higher than the corner voltage. The first segment must naturally start from zero and it has the steepest slope.

The simplest solution would be to design an attenuator for sqrt(V_{IN}), and because sqrt(3)=1.73, amplify the result by 3/1.73. Then +3V input would yield +3V output, as de-

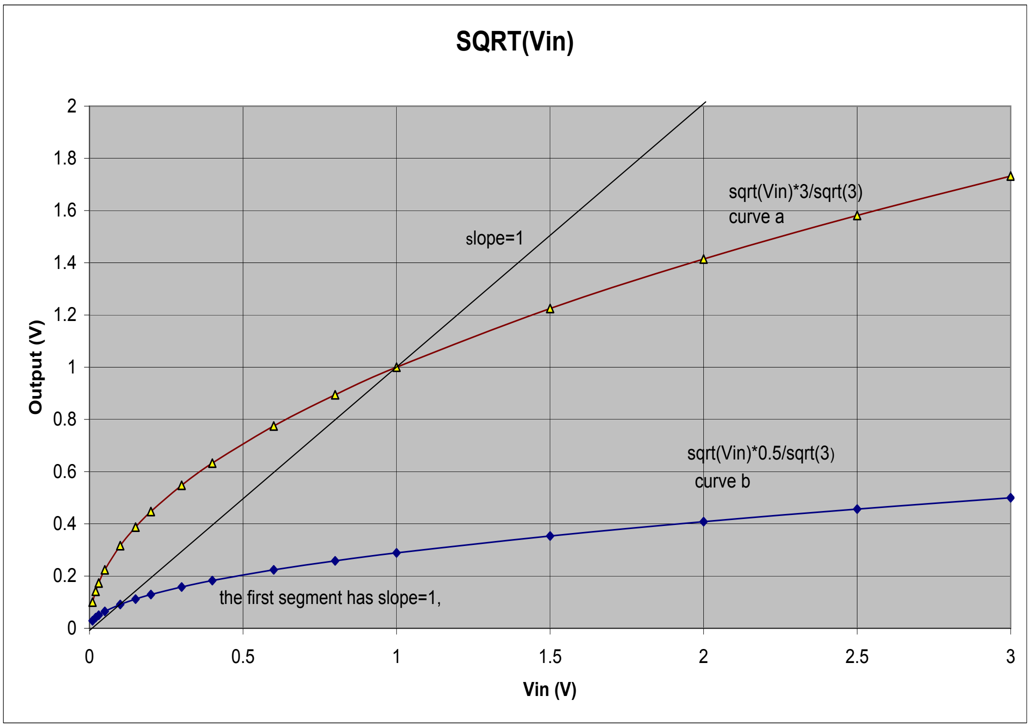

Fig. 1. Square root of V_{IN} and the same scaled down

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sired. But the steepest slope for an attenuator is 1 (= no attenuation). If a line with slope 1 is drawn from the origin, it can be seen that the error to the real sqrt function is very large, see Fig.1. This can be avoided by scaling down the square root so, that the maximum output for 3V input is much less than 1.73V. The scaling factor is a compromise between accuracy of the first segment and offsets of the op amp inputs. We chose 0.5V maximum output (Fig.1 curve b). The attenuator output must then be amplified by a factor of 6.

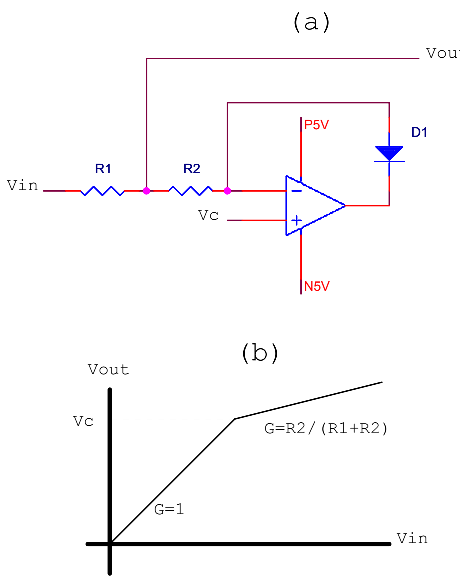

Fig. 2. Simple case of two segments

PRINCIPLE OF OPERATION

The idea is to generate zero-impedance nodes at selected corner voltages. Each node should exhibit infinite impedance for input voltages less than the corner voltage, and zero impedance for input voltages higher than the corner point. These nodes are used for building an attenuator, whose attenuation factor depends on the input voltage. Figure 2a shows the simplest case of only one corner, or two segments. As long as V_{IN} is less than V_C, the inverting input is more negative than the inverting input. The op amp output is near to the positive supply, diode D1 does not conduct and the right side of R2 sees a very high impedance. V_{OUT} equals V_{IN} because there is no attenuation. But as soon as V_{IN} exceeds the corner voltage V_C, the op amp output changes polarity. Diode D1 conducts and the op amp tries to keep its inverting terminal at V_C. If the input voltage still increases, the slope of the new growth will be R2/(R1+R2). See Fig. 2b.

Figure 3a shows the next step of having two corner points, and it also shows the principle of the final circuit. The two corner points are formed by a simple voltage divider from some reference voltage V_{REF} so that $V_{C1}<V_{C2}$. Selection of the corner points is a kind of matter of taste, where one tries to keep the approximation error tolerable within the range where V_{IN} is likely to vary. It is important to notice, that the corner points must be taken from the V_{OUT} axis for calculating the resistors for the V_C divider, as shown by Fig. 3b.

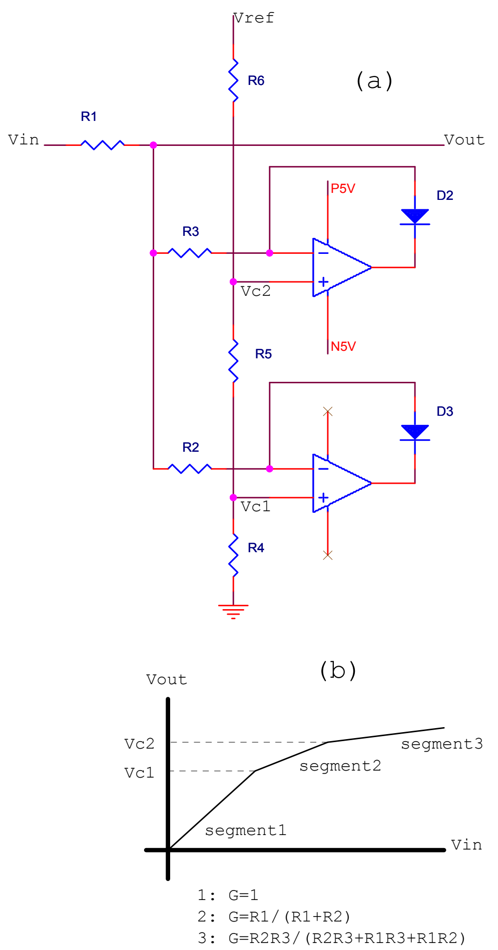

Fig. 3. Three segment approximation

Calculating resistor values for the attenuator is a little tedious. A lot of help is provided by formula

(2) $$V_{OUT} = (V_{IN}/R1 + V_{C1}/R2 + V_{C2}/R3 + \ldots)/(1/R1 + 1/R2 + 1/R3 + \ldots)$$

Using V_{IN} values corresponding to the corner points, we can solve first for R2, then for R3 and so on, if the circuit has more than two corner points. A spreadsheet program with goal seeking facility is good enough, unless one has to do this calculation frequently.

THE FINAL CIRCUIT

The function to approximate is (Fig.1 curve b)

(3) $V_{OUT} = \mathrm{sqrt}(V_{IN})*0.5/\mathrm{sqrt}(3)$

The first corner voltage cannot be selected, it is determined by the fact that the first segment starts from zero and its slope is one. The next three corners were chosen visually:

$V_{C1} = 0.0834$ for $V_{IN} = 0.0834$
$V_{C2} = 0.172$ for $V_{IN} = 0.35$
$V_{C3} = 0.263$ for $V_{IN} = 0.83$
$V_{C4} = 0.365$ for $V_{IN} = 1.6$
$V_{C5} = 0.5$ for $V_{IN} = 3$

Calculating resistor values for the V_C divider needs trial and error: one must seek for a combination where all resistors are near to easily available stock values, e.g. from the E96 series (1% metal film). With our stock values, we end up to R6..R10 values shown in Fig. 3.

For calculating the attenuator resistances R1..R5 we use eq. (2) successively, first for R1 and R2, then for R1, R2 and R3 and so on. Each obtained value is replaced by a suitable stock value before the next step in order to prevent errors from cumulating. If no stock value is found, one can try to change R1, but this requires a new start from the beginning. We ended up with values R1..R5 shown in Fig. 4.

The non-inverting amplifier with gain 6 is trivial. Figure 5. shows how this approximation performs.

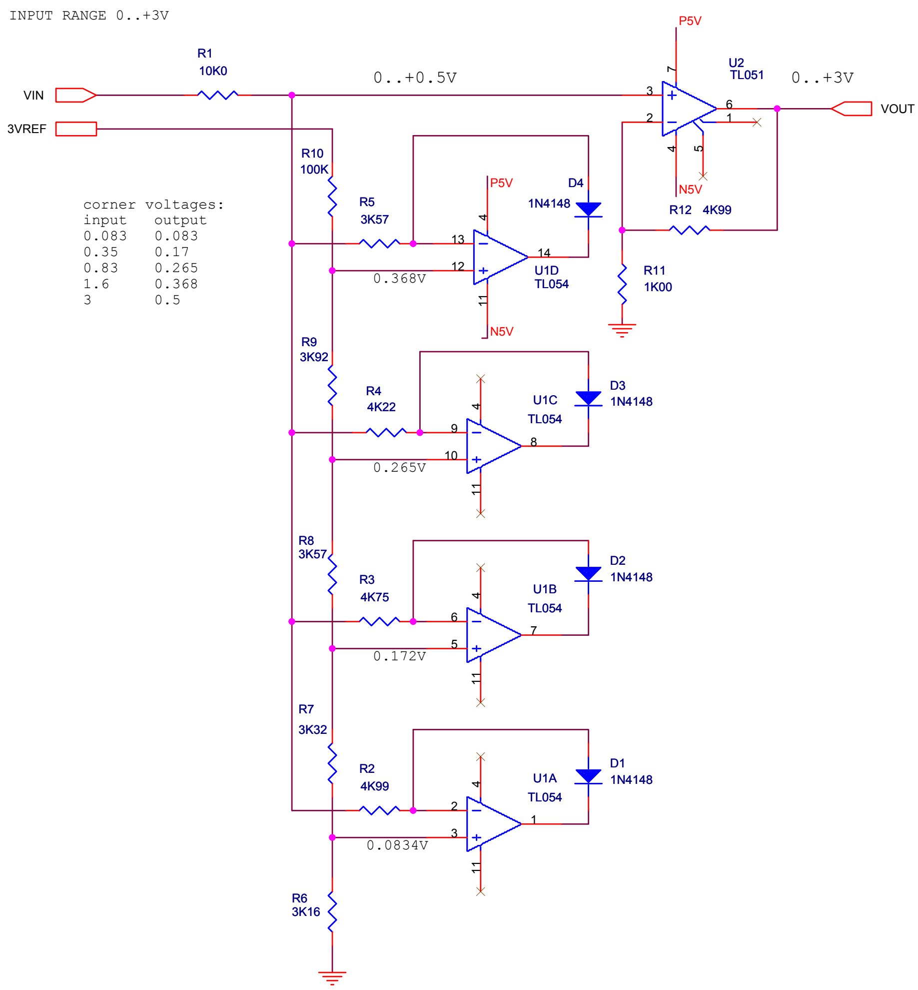

Fig. 4. The final circuit

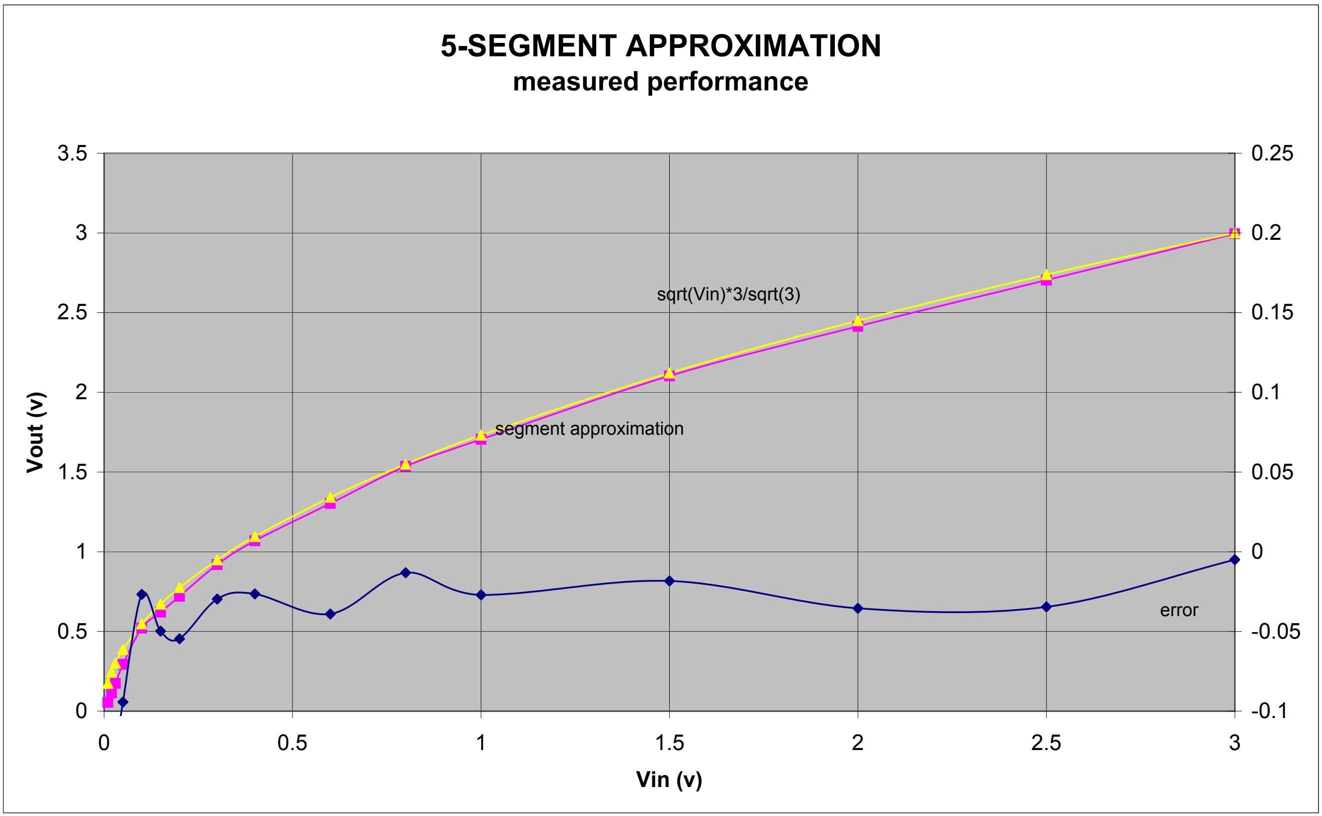

Fig. 5. Calculated (yellow) and approximated (red) square root.
The blue curve and y-axis on the right show approximation error.