IMPROVED ANALOG DIFFERENTIATOR (DERIVATOR) FOR THE AVS-48

Differentiator (derivator) is perhaps the most difficult-to-use term of a three-term (PID) controller. Unlike the proportional and integral terms, derivation is not necessary for an accurate control system. Its main function is to speed up settling after a change in the set point or after a change in other conditions like heat load in a temperature control system. If the set point is changed continuously and linearly (ramp), a properly tuned differentiator can reduce the error between the set point and the measured quantity. In the case of the **AVS-48 Picobridge®**, this is the resistance of the control sensor (Picobridge is registered trade mark of RV-Elektroniikka Oy).

SIMPLIFIED CIRCUITS

Fig. 1. shows the basic differentiator. Its output is

$$V_O(s)=\frac{-R_2}{\frac{1}{C_1*s}}*V_I(s)=-C_1R_2*s*V_I(s)$$
$$V_O(\omega)=-C_1R_2*\omega*j*V_I(\omega)=G_D*\omega*j*V_I(\omega)$$

where s is the Laplace variable, ω is the angular frequency and j is the imaginary unit. The C_1R_2 is the derivator time (it is given in seconds) but we will often call it incorrectly “derivator gain” for reasons that are explained later. The main drawback of this ideal derivator is, that its gain grows to infinity with the input frequency. This can make the output very noisy. In a cryogenic temperature control system based on a resistive control sensor, only a very low excitation power is allowed, and the signal will usually contain significant amounts of noise. Changes in the controlled temperature are usually quite slow, and the derivator needs not be effective above a few Hz.

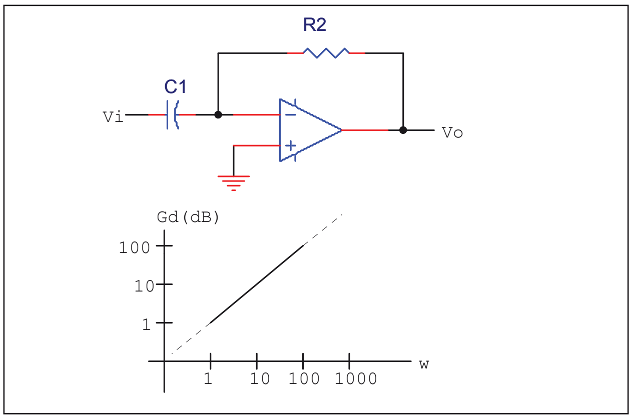

Fig. 1: The basic differentiator

Fig.2. shows the first modification to the ideal differentiator. At angular frequency $\omega=1/C_1R_1$, its gain stops growing and stays at $-R_2/R_1$ for all higher frequencies. This prevents the gain from rising to infinity with increasing frequency. Unfortunately, if we want to keep high-frequency amplification low, also the derivation effect will remain low.

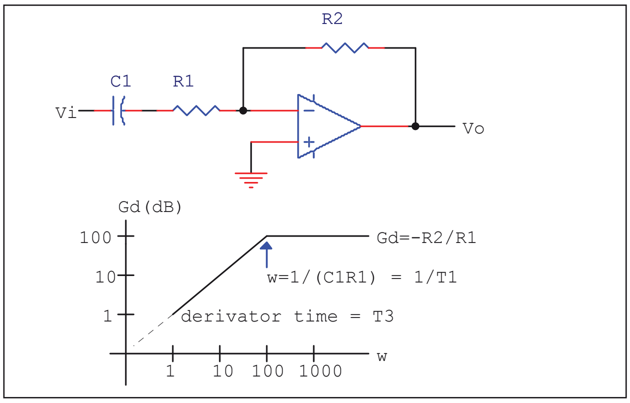

Fig. 2: The basic differentiator with limited high-frequency gain

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Fig. 3. is the simplified circuit of the differentiator used in the AVS-48. The capacitor C_2 has been added to the circuit of Fig. 2. in order to further reduce amplification at high frequencies, and a second amplifier stage compensates for the attenuation of the derivation circuit. Component selection for this circuit is somewhat tricky, as it tends to be for many sophisticated analog circuits.

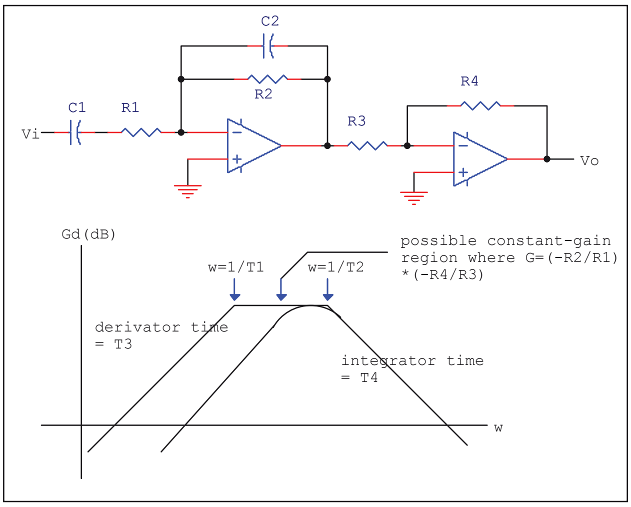

Fig. 3: The basic differentiator has been improved for the AVS-48 by adding C_2 and an inverting amplifier stage.

We define four time constants for simpler notation

$$\tau_1 = C_1R_1$$
$$\tau_2 = C_2R_2$$
$$\tau_3 = C_1R_2$$
$$\tau_4 = C_2R_1$$

Straightforward calculation of the overall gain yields

$$V_O(s) = \frac{-\tau_3 * s}{(1+\tau_1 * s)(1+\tau_2 * s)} * \left(\frac{-R4}{R3}\right) * V_I(s)$$

and for sinusoidal input,

$$V_O(\omega) = \frac{-\tau_3 * \omega * j}{(1+\tau_1 * \omega * j)(1+\tau_2 * \omega * j)} * \left(\frac{-R4}{R3}\right) * V_I(\omega)$$

This results in behaviour resembling that shown in Fig. 3. At very low angular frequencies both τ_1*ω and τ_2*ω are small compared with 1, and therefore

$$V_O(low\ \omega) \approx -\tau_3 * \omega * j * \left(\frac{-R4}{R3}\right) * V_I(\omega)$$

which is an ideal derivator's output with derivation time τ_3 (or "derivation gain" $G_D=\tau_3$).
At very high frequencies, both τ_1*ω and τ_2*ω are large compared with 1, and then

$$V_O(high\ \omega) \approx \frac{-\tau 3 * \omega * j}{(\tau_1 * \omega * j)(\tau_2 * \omega * j)} * \left(\frac{-R4}{R3}\right) * V_I(\omega) =$$
$$= \frac{-1}{C_2R_1 * \omega * j} * \left(\frac{-R4}{R3}\right) * V_I(\omega) = \frac{-1}{\tau_4 * \omega * j} * \left(\frac{-R4}{R3}\right) * V_I(\omega)$$

which is an integrator's output with integration time τ_4 (or "integrator gain" $G_I= \tau_4$).

Between these two extremes and depending on component selection, we have a more or less wide region where τ_1*ω is higher than 1 but τ_2 * ω remains lower than 1. The gain can be approximated as

$$V_O(middle\ \omega) \approx \frac{-\tau_3}{\tau_1} * \left(\frac{-R4}{R3}\right) * V_I(\omega) = \left(\frac{-R_2}{R_1}\right) * \left(\frac{-R4}{R3}\right) * V_I(\omega)$$

Within this center region, if it is wide, the circuit exhibits frequency independent gain, and this is one reason why we use the incorrect terms "derivator gain" and "integrator gain", although they are expressed in seconds. (Operating convenience: Increasing derivation gain means more effective derivation and increasing integrator gain means more effective or faster integration). τ1 must be ≥ τ2, but if they are equal or near to each other, the middle range shrinks to non-visible and also the gain remains lower than R_2R_4 / R_1R_3 (see Fig. 5.)

COMPONENT SELECTION

After a succesful selection of component values, the modified derivator of Fig.3. would be suitable for an application where the environment and control parameters remain constant.

That is not the case in cryogenic temperature control, where heat loads, thermal conductivities and heat capacities can vary within an order or orders of magnitude. A general-purpose temperature controller, like that in the **AVS-48 Picobridge®**, must offer a wide range of adjustable or selectable parameters, from which the user can choose the ones that work best in the present application and temperature.

There are four different time constants that depend on four component values. None of the time constants is totally independent, each component value appears in two time constants. In addition, there is the additional inverting gain stage. We must make some decisions or wishes in order to keep the task manageable.

- It is more convenient and needs less real estate on the pcb to switch resistors than capacitors, so we fix C_1 to be the largest easily available polyester film capacitor, 3.3μF, and make R_1 selectable.

- The derivator should not amplify 50Hz mains hum regardless of which derivator gain the user has selected.

- The derivator's purpose is to increase the control system's gain but no more than needed for speeding up settling. If too much gain is added, the system starts to oscillate. Therefore we concluded -quite arbitrarily- that the derivator should never increase the gain by more than 30dB.

- We make only two resistor values selectable, namely R_1 and R_4.

By switching R_1, we can select τ_1. The inverting gain needs to be specific for every setting, that is why we must switch also R_4. After some trial and error, the following fixed values were "guessed": C_2 = 1μF, and R_2 = 100 kΩ and R_3 = 1 Ω. R_1 and R_4 are selected by analog switches (CMOS 4051). An Excel worksheet was used for finding manually 10 logarithmically spaced combinations of R_1 and R_4 so, that at least 10dB gain increase can be achieved for frequencies from 0.001Hz to 1Hz, which is believed to cover the frequency range ever encountered in cryostat control.

THE COMPLETE SCHEMATIC

The derivator part of the complete circuit (Fig. 4) is formed by U_1 and analog switches U_2 and U_3. The derivator gain is selected by a 4-bit byte DGAIN. If DGAIN is zero, the derivator is disabled. Note that the gain-determining resistors are all within a reasonably convenient range from 3 kΩ to 3 MΩ (this affected selection of R_2 and C_2). The inverting stage, formed by U_4, U_5 and U_6, is also controlled by DGAIN. It has ten selectable gains and gain -1 for the disabled position, when DGAIN is 0. The offset adjustment is useful, as low-cost op amps have offset voltages of a few millivolts, which may be amplified by 1000. The feedback resistors fall within the convenient range from 1 kΩ to 1 MΩ.

The CMOS circuits work with VEE=-5V and VDD=+5V. Their inputs need not be protected against overvoltage, because also the operational amplifiers use +/- 5V power supplies.

The derivator response to a sinusoidal input is shown in Fig. 5. The ten curves, corresponding to ten derivator gains, are separated by a rough 5dB up to 0.02Hz. Above this frequency, the highest gains saturate to 30dB which ensures, that the circuit has a limited amplification for noise or other interference, whose frequency is above our interest. Above 10Hz, all curves converge toward an integrator so that 50Hz mains hum is not amplified (0dB).

The curves represent the magnitude of the response. The phase angle has not been calculated, but it must change from the +90 degrees of an ideal derivator to -90 degrees of an ideal integrator.

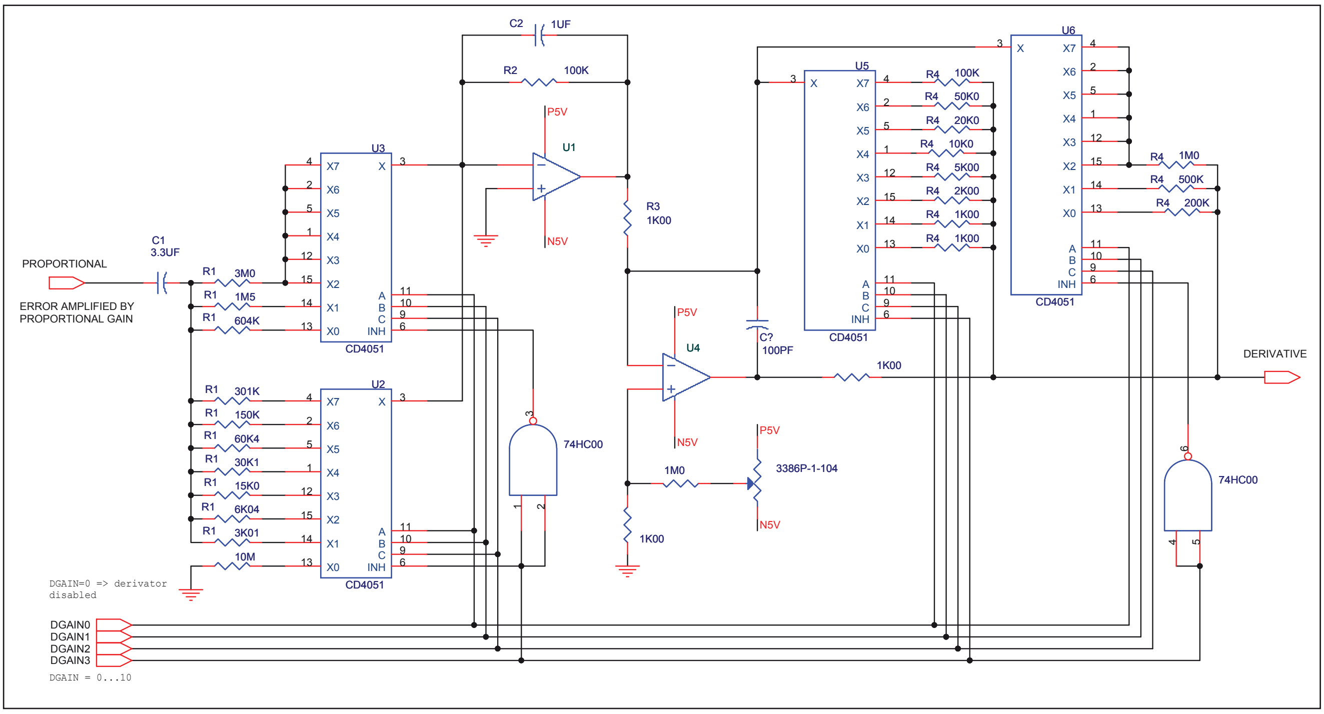

Fig. 4: The complete schematic of the AVS-48 differentiator

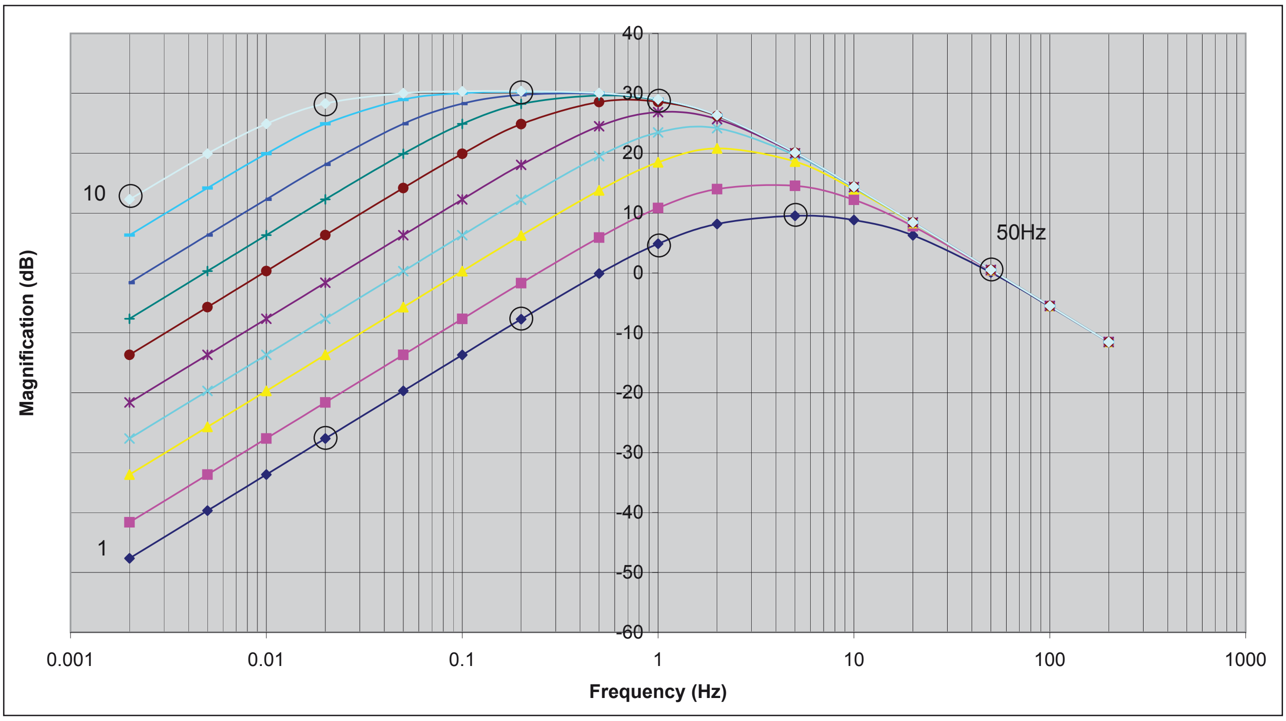

Fig. 5: Calculated response for sinusoidal input. Circles indicate points that were verified by measurement.